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KINETIC EQUATIONS FOR SUSPENSION SEPARATION

I. A. Vainshtein

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A kinetic equation for description of suspension separation is proposed. Examples of determining effective rheological characteristics and phase separation surfaces during filtration are presented.

At the present time, various models are used to describe processes occurring in separation of suspensions — motion of a liquid through a porous layer of variable thickness with constant (or variable) porosity, motion through a set of capillaries which decrease in number (or diameter), or motion of particles in a suspension, precipitate, or filter pores, etc. [1-4].

The existing models are imperfect: several times deviations have been noted in theoretical expressions based on Darcy's law, the Hagen—Poiseuille or Stokes equations, etc., as compared to experimental data obtained by separation of real suspensions [3-6]; a number of modified equations and empirical expressions have been proposed [1-5, 7, 8]. Moreover, the differing dimensions of the parameters appearing in the equations do not permit comparison of suspension properties which determine the selection and productivity of equipment to be used. Thus, the basic characteristic of filtration with formation of a precipitate — specific resistance — cannot be applied to filtration of the very same suspension if the filter barrier pores are comparable in size to the dimensions of the precipitate particles and filtration occurs with the pore sealing process, etc.

The unsatisfactory state of suspension separation theory and technology has been evaluated several times [9, 10]. In reality, as Romankov noted, all suspensions are non-Newtonian liquids, and a theory of their separation should be developed from universal concepts [11].

One reason for the lack of a unified mathematical description of suspension separation is the insufficient development of theory in the rheology of two-phase flows in general, despite the large number of studies dedicated to this problem [12, 13]. The complexity of the phase interaction mechanism, especially when physicochemical processes on the interphase boundary are considered, eliminates the possibility of the early appearance of a unified description and compels us to seek new and better semiempirical models.

Below we will present a model, according to which as the suspension separates and the volume of liquid phase in the two-phase system decreases, there occurs a corresponding reduction in the mean distance between solid phase particles.

We assume that the suspension is isotropic, that external forces in separation are compensated by friction forces, that inertial forces are absent, and that motion is one-dimensional and laminar, and we will neglect the effect of the vessel (filter) walls. As is well known, due to phase interaction in the surface layer, the apparent viscosity of the suspension differs from the dynamic viscosity of the liquid, a fact which is considered by a quadratic dependence

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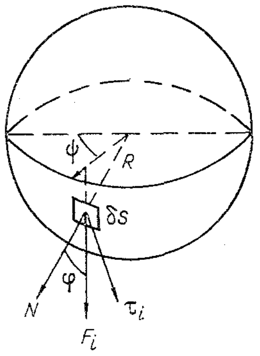

Fig. 1

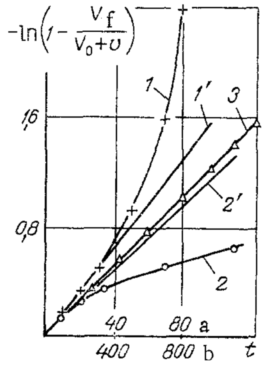

Fig. 2

Fig. 1. For derivation of Eq. (3).

Fig. 2. Filtration kinetics of suspensions with various properties: 1-3) experimental data ($v = 0$); 1, 3) scale a (sec); 2) scale b (sec); 1', 2') "rectified" curves 1, 2, at $v/V_0 = 0.12$ and 0.19.

$$\eta/\eta_l = 1 + a\mathrm{v} + b\mathrm{v}^2. \tag{1}$$

For suspensions we may use the rheological equation of state of a non-Newtonian liquid, which is often written in the form

$$\tau = \eta\dot{\gamma}^n. \tag{2}$$

With the assumptions made above, the resistance to deformation τ_i of an elementary liquid area on the phase boundary δS can be equated to the tangential component of the external force F_i, acting on this surface element δS (Fig. 1):

$$\tau_i = \frac{F_i}{\delta S}\sin\varphi. \tag{3}$$

We assume that the suspension contains spherical particles of radius R. Then the ratio $F_i/\delta S$ can be replaced by F/S. The mean value τ_m in the layer bounding the particle is defined by the expression

$$\tau_m = \frac{1}{4\pi R^2}\frac{F}{S}\int_0^{2\pi}\int_0^{\pi} R^2\sin^2\varphi d\varphi d\psi = \frac{\pi F}{4S}. \tag{3'}$$

Equation (3') is valid not only with respect to the liquid volume surrounding a particle, but also with respect to any portion of the suspension volume, including the entire volume, if the concentration gradient may be neglected. It is also valid for suspensions with particles of arbitrary form and size if the suspension may be considered isotropic.

Assuming its uniformity, the shear rate may be taken as the gradient of the mean (over volume) flow velocity

$$\dot{\gamma} = \frac{dU}{dH} \cong \frac{U_m}{H_m}. \tag{4}$$

Here $U_m = -(1/2)dV/sdt$; $H_m = V/S$.

With consideration of the assumptions made, after substitution of the averages over volume in Eq. (2) and various transformations we have

$$\frac{dV}{V}(1 + a'\mathrm{v} + b'\mathrm{v}^2) = -Adt, \tag{5}$$

where

$$A = \left(\frac{\pi sF}{\eta_l}\right)^m \frac{1}{2^{2m-1}S^{m+1}}; \; m = \frac{1}{n}; \; a' \cong ma; \tag{5'}$$

$$b' \cong mb + \frac{m(m-1)}{2}a^2.$$

For suspension separation by filtration $F_f = s\Delta P$; for separation by precipitation, according to [2, p. 25], $F_s = V_s g \Delta\rho$.

Integrating Eq. (5) for the conditions of a periodic process at F = const from t_0 to t and from V_0 to V, after simple transformations we write

$$\ln\frac{V}{V_0} + (a' + b')\ln\frac{V(V_0 + V_s)}{V_0(V + V_s)} + b'\left(\frac{V_0}{V_0 + V_s} - \frac{V}{V + V_s}\right) = -A(t - t_0). \tag{6}$$

In order to determine A, a', and b', it is sufficient to have three pairs of V and t values. However, aside from the known parameters, A contains the two unknowns S and n. To obtain a second equation we use the concept of "wall" effect — "slippage" or "braking" upon motion of the liquid relative to the solid phase [14]. With consideration of these phenomena, in the above expressions we take a = b = 0, n = 1. Then Eq. (6) takes on the simpler form:

$$\ln\frac{V + v}{V_0 + v} = \ln\left(1 - \frac{V_f}{V_0 + v}\right) = -A'(t - t_0), \tag{6'}$$

where $A' = \pi sF/2S^2\eta_l$.

All suspensions are one of three types: anomalous at $v < 0$ (corresponding to a braking wall effect) or $v > 0$ (slippage) or Newtonian at $v = 0$ [14].

In suspensions of the first type, $v < 0$. In such systems a portion of the liquid is kinetically bound to the dispersed medium (enters into closed pores, is maintained by surface forces, is immobilized by spatial structure, etc.), while the remaining portion has normal viscosity. A characteristic parameter of suspensions of this type is v/S, the mean thickness of the liquid surface film. The anomaly may have another cause — an increase in phase boundary surface due to, e.g., destruction of aggregates. The characteristic of a suspension of this subtype would be S/S_0 or $(V + v)/V$.

A value of $v > 0$ corresponds to a suspension with properties opposite those of the first type. For example, the effective phase-boundary surface increases due to formation of more-or-less durable aggregates when attractive forces dominate over repulsive ones as particles approach each other. A different subtype of this class of suspensions is dispersed structural systems, in which as shear and destruction of the spatial structural cell increase, a portion of the dispersed medium is freed and fluidity increases.

In the third class of suspensions ($v = 0$) we have those in which either no phenomena occur on the phase boundary, or the aggregates formed upon approach of particles are inclined to flow at low shear stress.

In more complex systems, when because of phase interaction (sorption—desorption of surface-active materials) the nature of the anomaly changes or manifests itself only at certain times, there should be corresponding changes in the parameter v.

Thus, independent of the nature of the physicochemical processes occurring on the phase boundary as the phases separate, anomalies can be considered by the parameter v, which is functionally related to the rheological characteristics of the dispersed system.

We will present some examples of the use of Eqs. (6) and (6') to determine the filtration and rheological characteristics of suspensions obtained by various means from hydroxide and salt solutions combined with magnetite, gypsum, and mixtures thereof [15, 16]. Using the experimentally determined dependence of filtrate volume on time, we construct a graph in the coordinates $\ln(1 - [V_f/(V_0 + v)]-t$ and use the slope of the line (at $v \neq 0$ after "rectification" of the curves with simultaneous calculation of v by graphical or analytic methods (Fig. 2)) to determine A', and then S and S_{sp} (see Table 1).

TABLE 1. Suspension Filtration Parameters

Expt. No.	Precipitate composition	s, m^2	ΔP, MPa	A', sec^{-1}	S_{sp}, m^2/g	v/V_0	r (10^{11}), m/kg
1	Fe-III hydroxide, gypsum	$26\cdot10^{-4}$	0,1	16,2	49	0,12	3,4
2	Fe-III hydroxide	$26\cdot10^{-4}$	0,1	1,2	120	—0,19	34
3	»	$26\cdot10^{-4}$	0,1	12,7	230	0	170
4	Fe-II hydroxide	$26\cdot10^{-4}$	0,1	1,75	79	0	20
5	Fe-III hydroxide, gypsum	$26\cdot10^{-4}$	0,1	0,65	63	0,05	2,2
6	»	$1\cdot10^{-2}$	0,1	9,89	71	0	18
7	»	$1\cdot10^{-2}$	0,2	1,65	83	0	23
8	»	$1\cdot10^{-2}$	0,3	2,22	89	0	26
9	»	$1\cdot10^{-2}$	0,5	2,02	92	0	37
10	»	$1\cdot10^{-2}$	0,6	2,13	110	0	45
11	Fe-II hydroxide, magnetite, gypsum	25	0,9	1,02	39	0	4,8

Note: Experiments 1-3 correspond to curves 1-3 of Fig. 2; experiments 6-10 are for steel corrosion precipitate produced by sewer water; experiment 11 is the same, with water removal by FPAKM-25 filter press [16].

Eliminating t from Eqs. (6) and (6'), we obtain a system of linear equations

$$(a'+b')\ln\frac{V_j(V_0+V_s)}{V_0(V_j+V_s)}+b'\left(\frac{V_s}{V_0+V_s}-\frac{V_s}{V_j+V_s}\right)-c\ln\frac{V_j+v}{V_0+v}=-\ln\frac{V_j}{V_0}, \tag{7}$$

where V_j is the value of $V(t)$ at $t = t_j$ ($j = 1, 2, 3$); $c = A/A'$.

Using this system we determine a', b', and c, and then from the relationship $c = (\pi s\Delta P)/4S\eta_l)^{m-1}$ we find n. Knowing a' and b' and employing Eq. (1), we calculate $\eta_{rel} = \eta(t)/\eta(t')$ for any time interval (t, t'), where $t_1 \leqslant t < t' \leqslant t_3$. For example, in the interval over which V/V_0 changes from 0.9 to 0.5 the following experimental data were obtained: at a value $v/V_0 = 0.12$ (Fe-III hydroxide—gypsum mixture, curve 1, Fig. 2), $\eta_{rel} = 0.58$, $n = 1.0003$; at $v/V_0 = -0.19$ (Fe-III hydroxide suspension, curve 2, Fig. 2), $\eta_{rel} = 1.81$, $n = 0.975$. For a number of suspensions $v/V_0 = 1$, $\eta_{rel} = 1$, $n = 1$ (curve 3, Fig. 2).

Apparently, the reason for viscosity decrease in the first experiment is a reduction in effective phase-boundary surface due to aggregation (or orientation) of the crystalline component of the dispersed phase—the gypsum ($S/S_0 = 0.87$). In the second case, considering a priori information on the properties of Fe-III hydroxide, one can propose the existence of a stable surface film ($v/S = 1.1\cdot10^{-6}$ m). It is natural that other methods of study do not give reliable information on the effective phase-boundary surface and the processes occurring during filtration due to unavoidable changes occurring during specimen preparation.

The effect of various process factors on the values of S and v was established: for example, with increase in ΔP, S_{sp} increases (see Table 1, experiments 5-10). Consideration of such relationships, which are related to the nature of phase conversions and surface phenomena, is beyond the scope of the present study.

We will consider the relationship between Eq. (6') obtained here and other known filtration equations. For experimental determination of filtration parameters, the corresponding equations are written as linear functions [3-8]. For example, for filtration with complete filling of pores:

$$W = W_{in} - k'V_f, \tag{8}$$

or with formation of a precipitate

$$\frac{k''V_f}{2} = \frac{t}{V_f} - \frac{1}{W_{in}}. \tag{9}$$

With consideration of the fact that $W = dV_f/sdt$, after integrating Eq. (8) from 0 to t and from 0 to V_f we obtain:

$$\ln\left(1-\frac{k'V_f}{sW_{in}}\right)=-k't. \quad (8')$$

Comparing this expression with Eq. (6'), we see that the function $V_f(t)$ is identical in the two cases. Consequently, Eq. (6') can be used not only to determine the filtration constant, since $k' = A'$, but also to determine S. In the given case (in filtration with precipitate formation this is less significant), by S we understand the effective phase-boundary surface, i.e., with consideration of the filtering barrier. Also, since in addition to A' we know υ (and thus S/S_0), it is simple to determine the value of the barrier and precipitate surfaces separately.

It is also evident from Eqs. (6'), (8') that $W_{in} = A'(V_0 + \upsilon)/s$. Similarly, comparison of Eq. (6') to Eq. (9) (the transformations necessary for the comparison involve expansion of the function in a series) shows that in this case also (and also in analysis of intermediate-type filtration equations) $W_{in} = A'(V_0 + \upsilon)/s$.

From this we may make the following conclusions. First, the various equations are not as different as they appear. Their generality has been suggested previously [17]. Aside from the above algebraic comparison, this is shown by the relatively small difference in constant values obtained by use of Eqs. (6') and (9). Thus, in the interval from $V_f/V_0 = 0.4$ to $V_f/V_0 = 0.9$, k''/A' varies by no more than 1.28 times, i.e., within the limits of accuracy of k'' determination from the straight portion of the curve.

The conclusion which follows, that the filtration rate is not a constant of the process, since it in turn depends on A', is confirmed by analysis of known filtration equations. Thus, at $t = 0$, $V_f = 0$, Eq. (8') becomes undefined. This same uncertainty appears in equations for filtration with complete pore filling, gradual pore filling, and intermediate-type regimes.

Our experimental data as well as those of [6, p. 10] indicate that W_{in} in fact depends on the precipitate properties, i.e., the relationship found between W_{in} and A' is confirmed.

Thus, the known models of the filtration process reflect a more general relationship (exactly for the case of pore filling, approximately for other regimes).

As is well known, the specific resistance of the precipitate is related to its porosity and to the phase separation surface, e.g., by the Kozeni—Karman equation. Considering that in the interval from $\varepsilon = 0.26$ to $\varepsilon = 0.8$, independent of particle form and size, the Kozeni constant is almost the same [18], by determining S with Eq. (6') we can also find the specific resistance of the precipitate at any level of compression in the filtration or pressing process.

NOTATION

A, A', suspension separation constants; a, a', b, b', rheological constants; F, force; g, acceleration of gravity; H, hydraulic radius; k, filtration constant; n, rheological constant; R, spherical particle radius; r, specific resistance of precipitate; S, phase-boundary surface; S_{sp}, specific phase boundary surface; s, precipitation or filtration boundary surface; t, time; t_0, period of nonstationary suspension separation; U, velocity; V, liquid phase volume; V_f, filtrate volume in receiver or volume of filtered layer; V_s, volume of solid phase; v, relative volume of solid phase; υ, parameter; W, filtration rate; γ, shear rate; ΔP, pressure head; $\Delta\rho$, density head; ε, porosity; η, effective suspension viscosity, η_l, dynamic liquid viscosity; τ, deformation; φ, ψ, angles (Fig. 1). Subscripts: 0, in, initial moment; i, value on surface element; j, value at certain moment in time; m, mean (over volume) value.

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COMBINING ASYMPTOTIC EXPANSIONS IN PROBLEMS OF THE FILTRATION OF A GAS—CONDENSATE MIXTURE

M. B. Panfilov

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The asymptote-combination method is used in the problem of the startup of a gas—condensate borehole in a porous bed, which is characterized as a singularly perturbed problem. An analytical solution is constructed.

Methods of singular perturbation, which have been intensely developed in recent years, offer great possibilities for the solution of nonlinear problems of filtration theory [1-4]. Primarily, this involves the problem of the filtration of a mixture of several fluids with phase transformations, when the process is determined by many factors, and their relative role differs in different regions of motion. The solutions of such problems include sharp transitions in narrow intervals (boundary layers).

In the present work, the possibilities of the combined-asymptote method in filtration theory are investigated for gas—condensate systems.

1. Formulation of the Problem of Startup of a Gas—Condensate Borehole. The process is investigated within the framework of a modified binary model [5, 6], in which it is assumed that the weight concentrations of the components in the liquid phase are constant

$$m\frac{\partial}{\partial t}[\rho_l s+\rho_g(1-s)]+\operatorname{div}(\rho_l\vec{V}_l+\rho_g\vec{V}_g)=0, \tag{1}$$

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